Eyes on Math

A visual approach to teaching math concepts

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power of visualization

- we use the power of visualization in math when we have students use manipulatives

visualization

- Communication in mathematics - creating a math talk community

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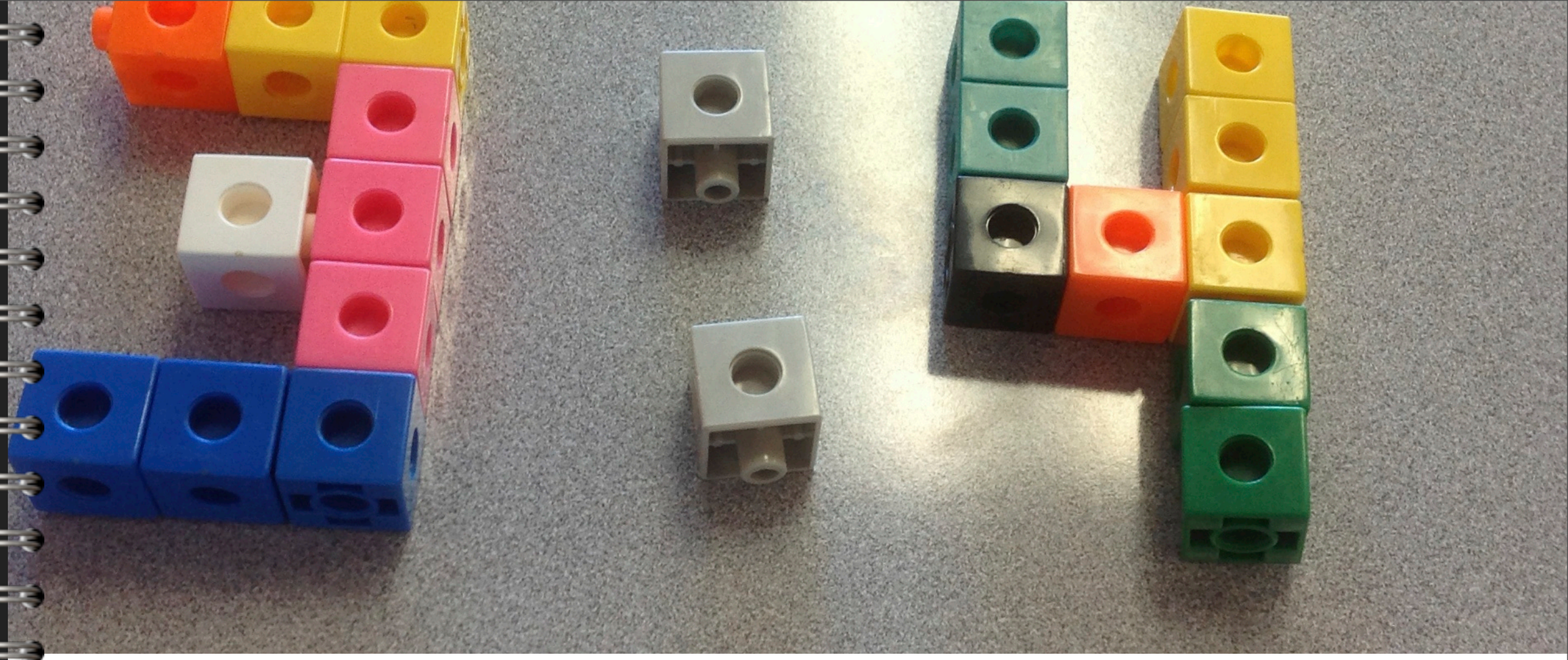
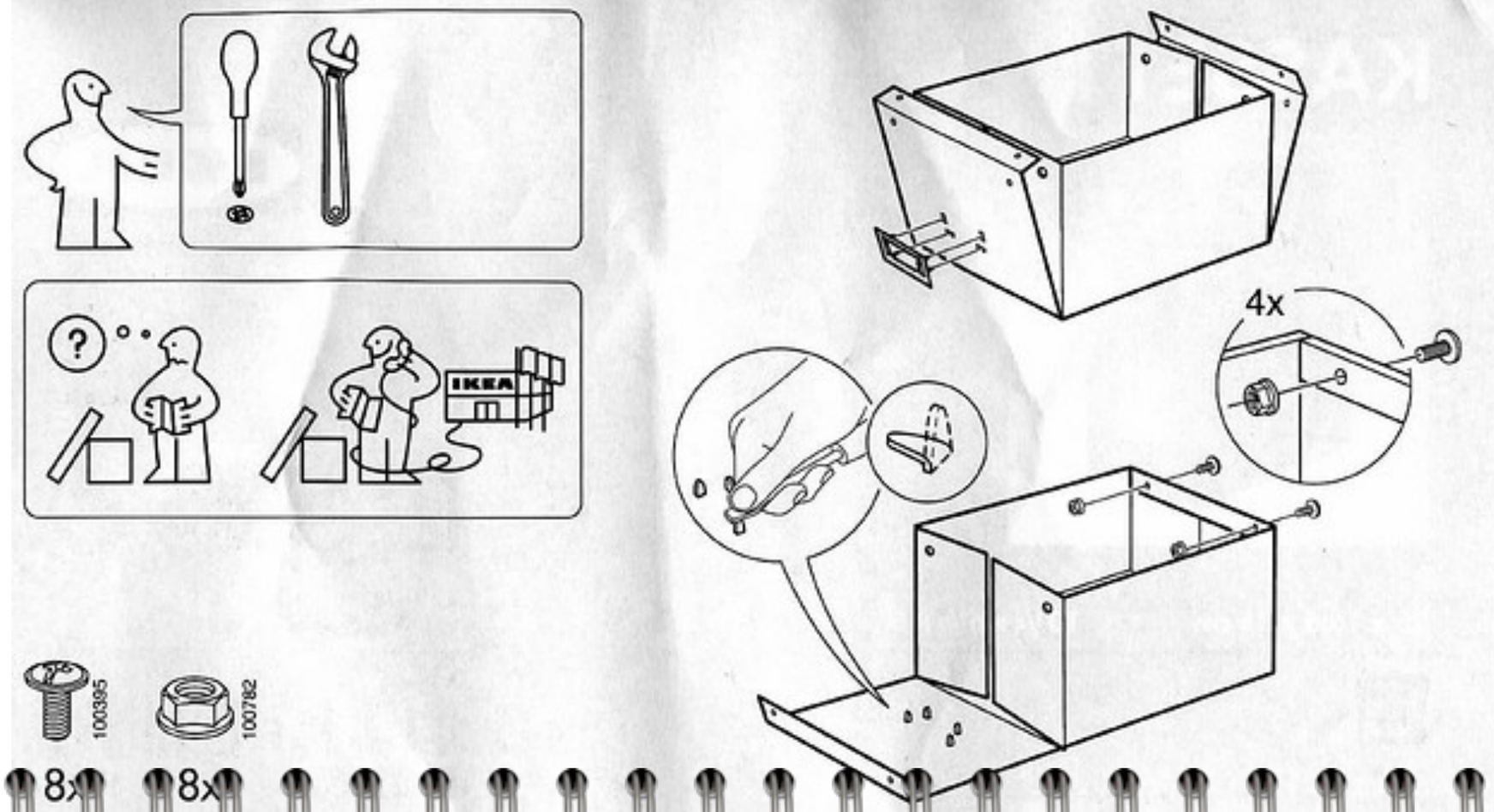


manipulatives

kinesthetic or visual?

picture books

make sense of math



we like visuals...

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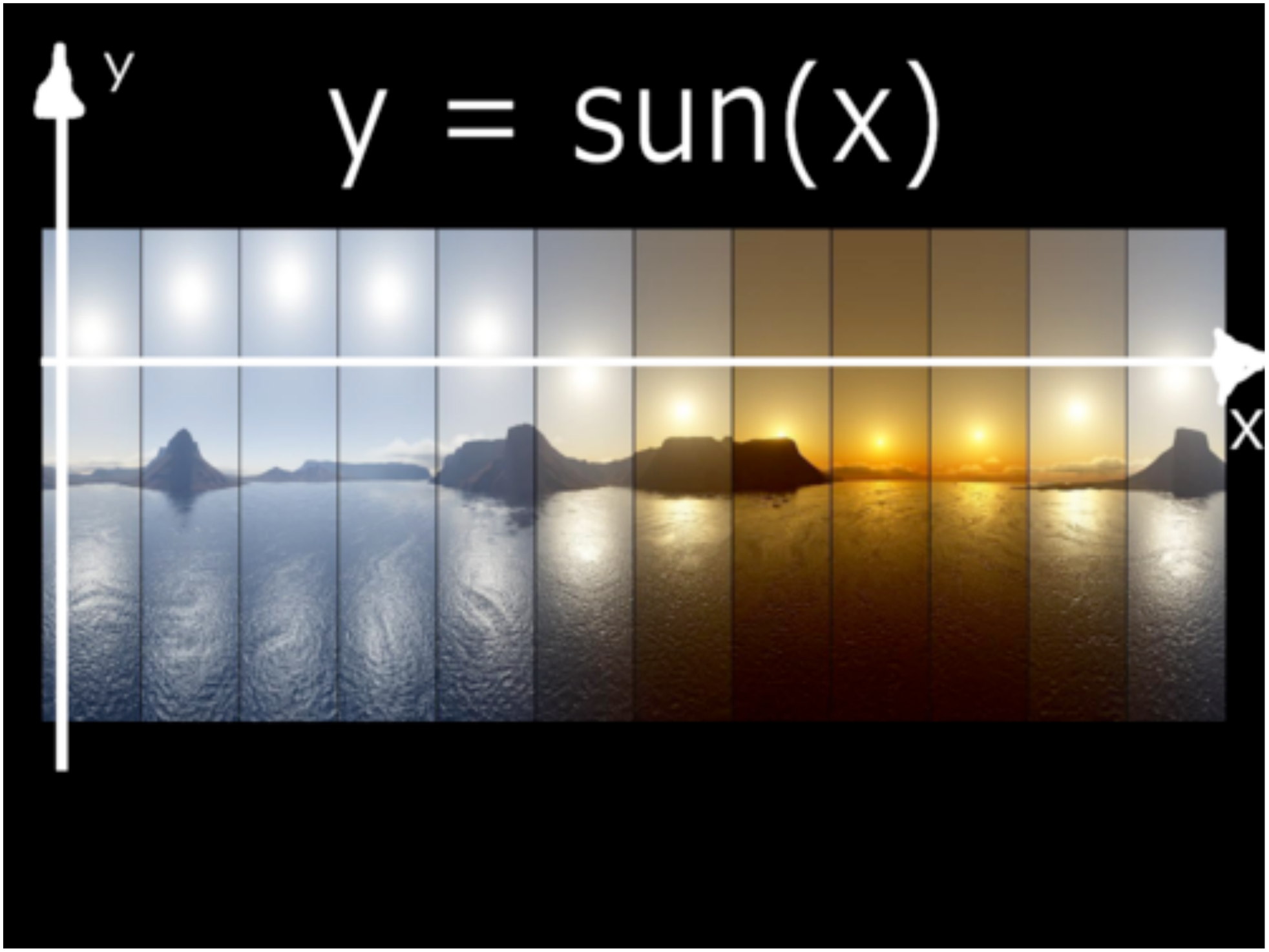
Gap Closing Strategy



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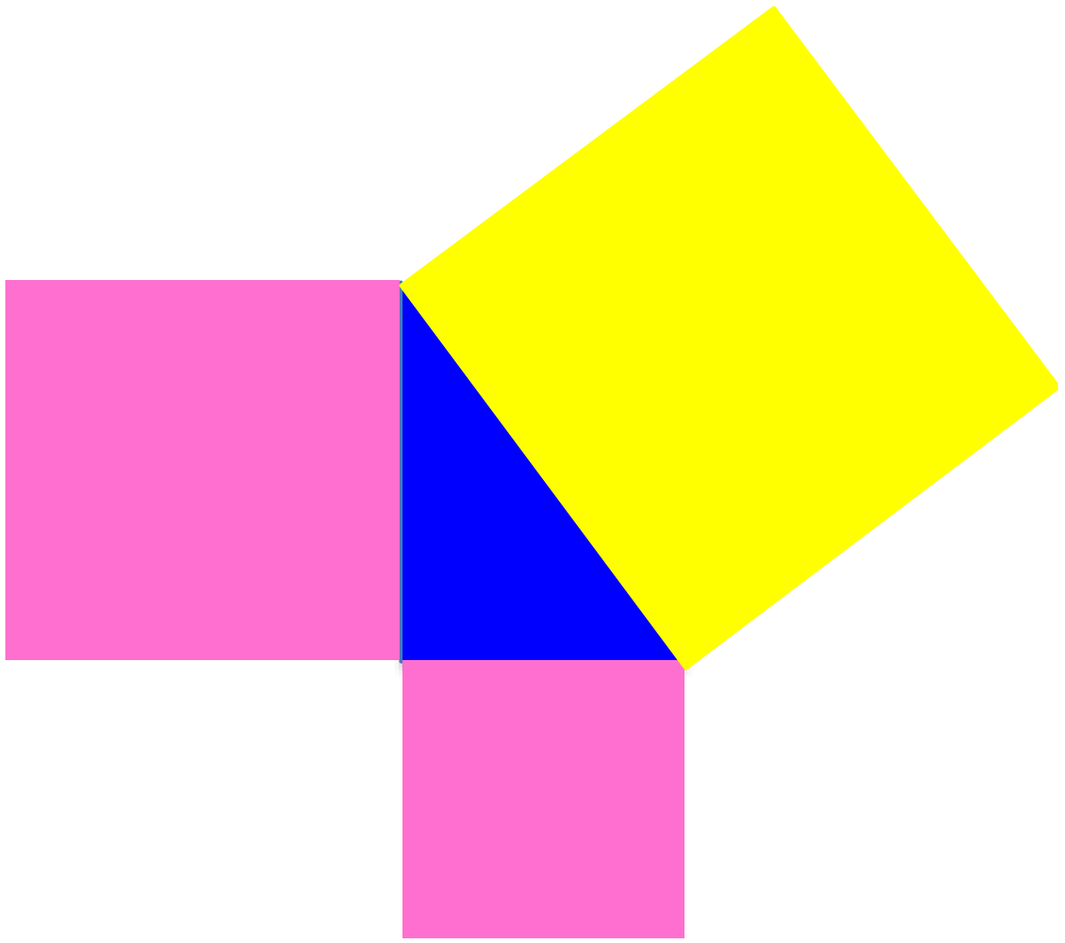
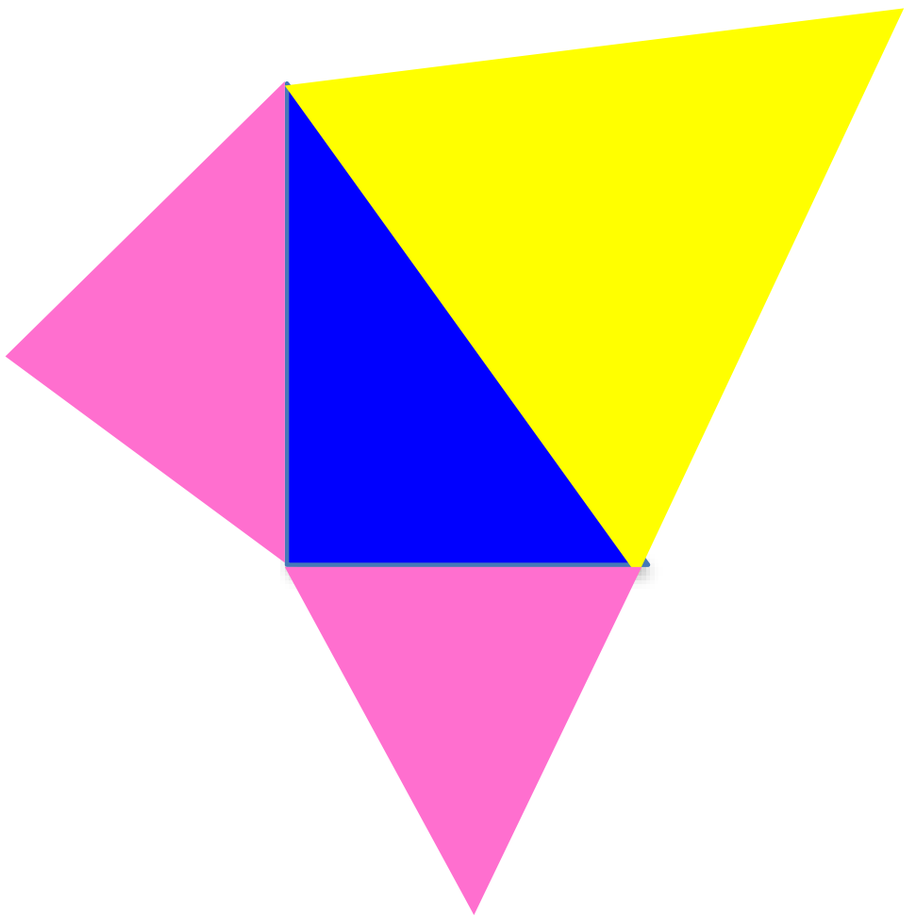
- So...let's use visualization for describing math concepts





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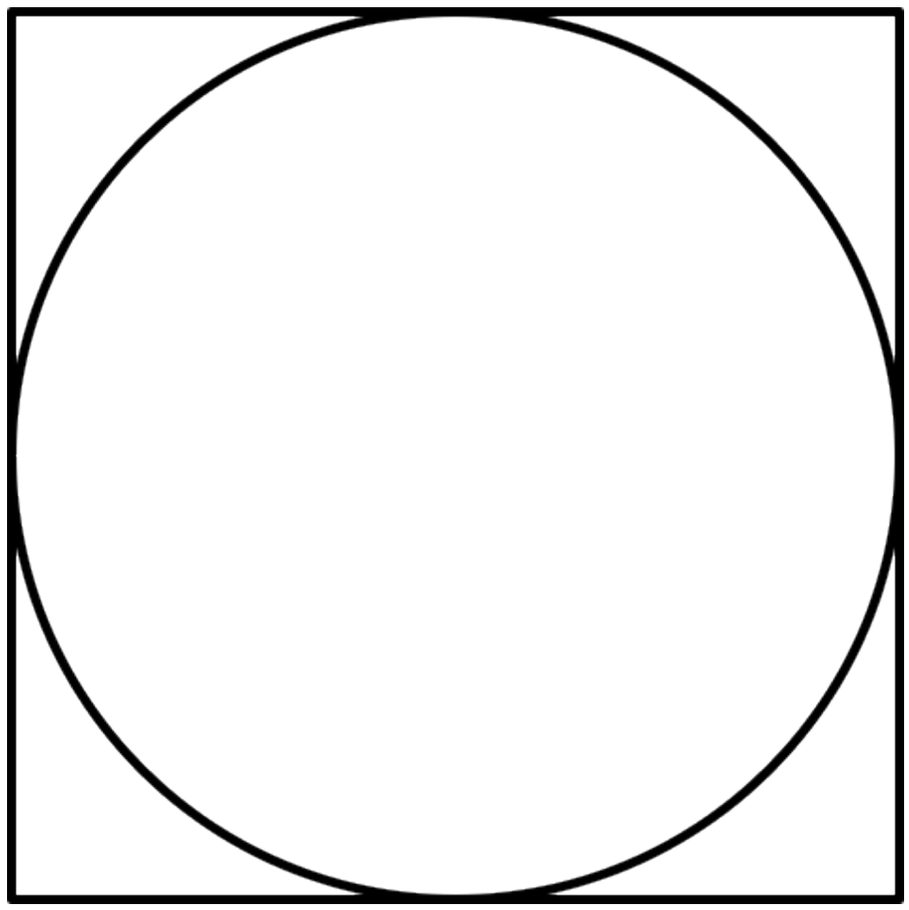
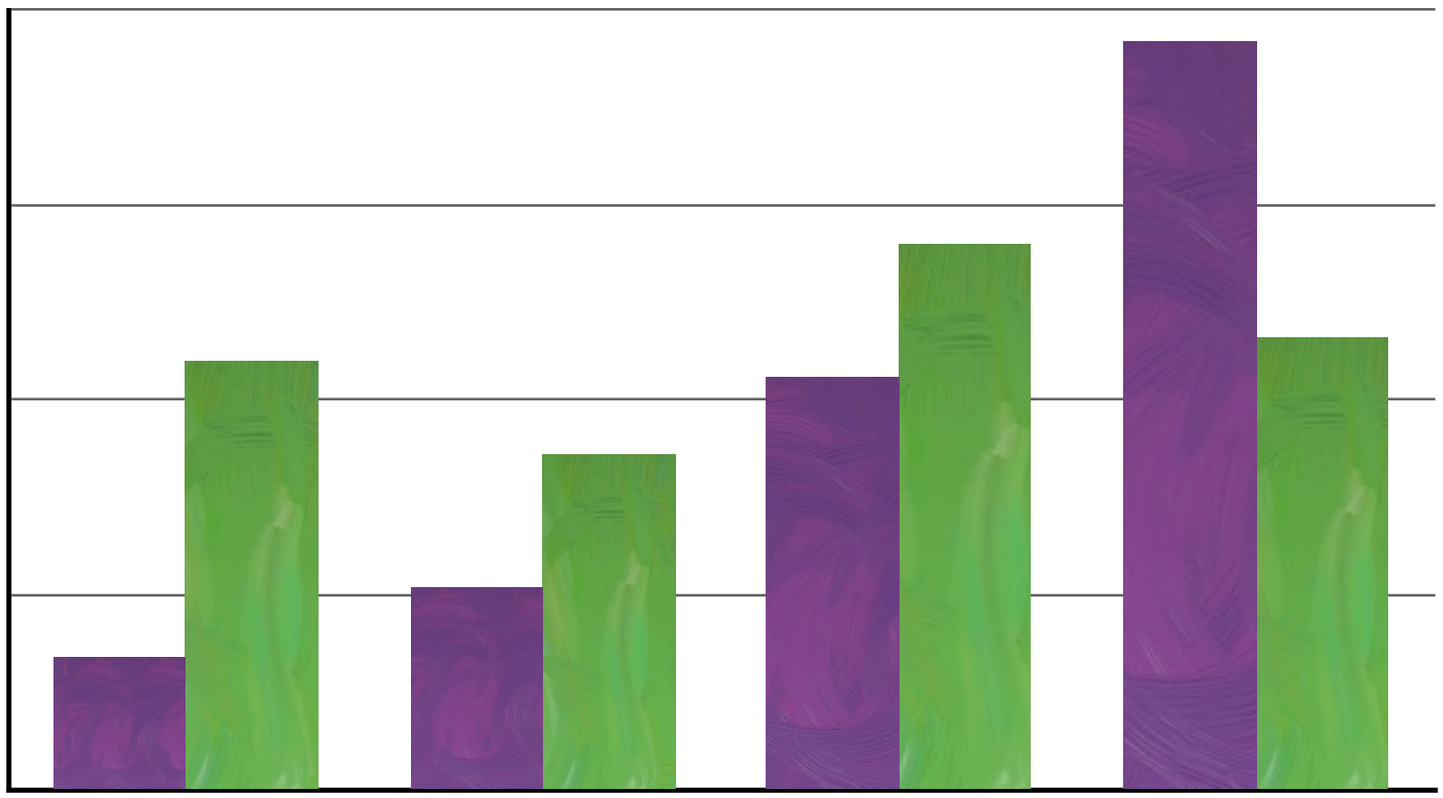


proof without words



open questions

use provocative visuals with open questions leading to rich conversations

Common Core Standards

- making sense of problems
- reasoning abstractly and quantitatively
- constructing viable arguments and critiquing the reasoning of others

Also...

- modelling with mathematics
- using appropriate tools strategically
- looking for and making use of structure

Can you write ___ x ___ to describe this picture?

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A picture about multiplication and how it is about equal groups. In the picture, most groups are equal - but notice that not all of them are. However, grouips of two can be rearranged to form grouips of 4 or the groups of four can be broken down into groups of 2. The picture is more interesting for students because it has an even number of groupis of two - which makes it possible to have both groups of 2 or 4.

questions

- When do you use multiplication?
- Are all the groups of penguins the same size? Does that matter when you are deciding if you can use multiplication?
- Could the penguins be rearranged into equal groups?

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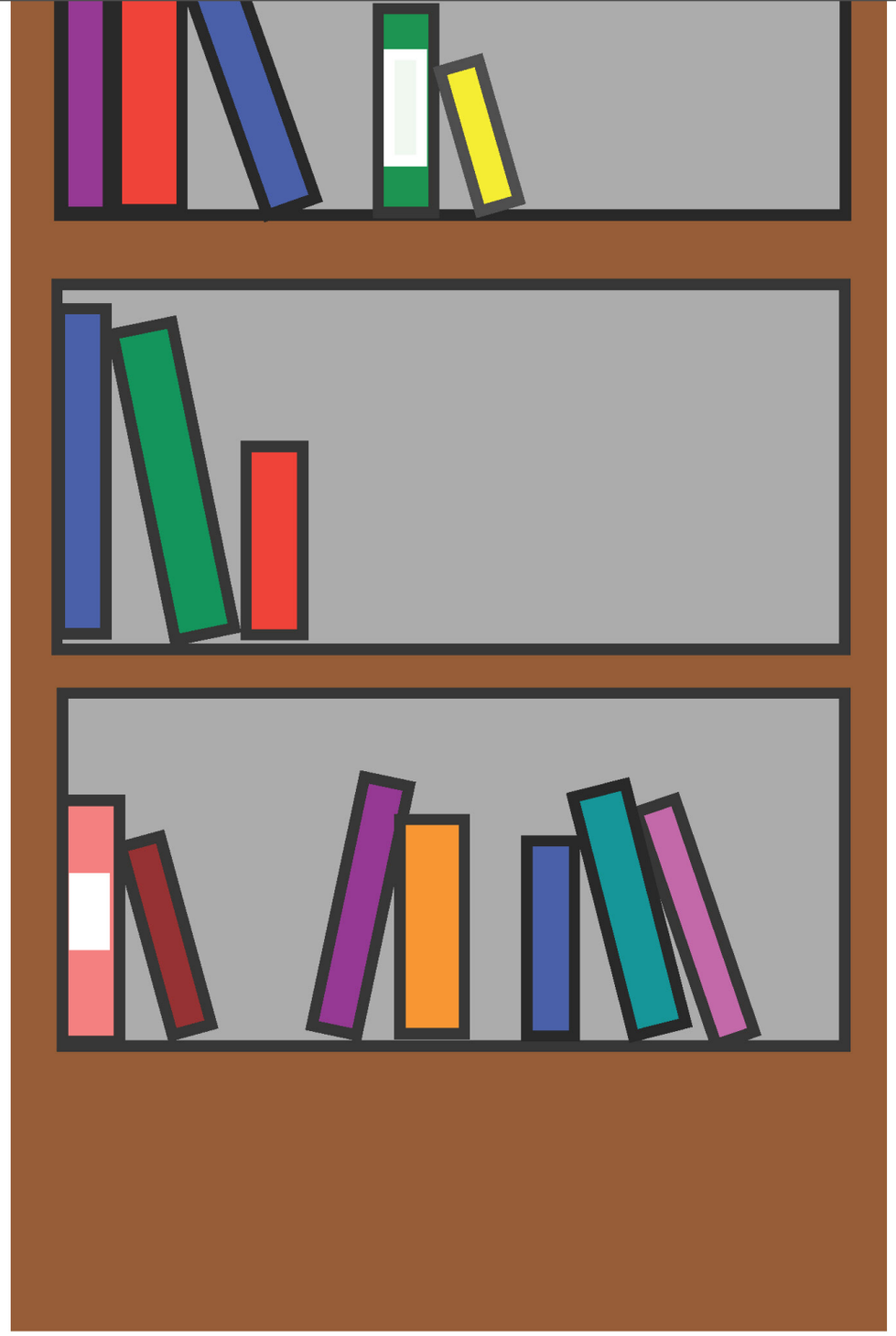
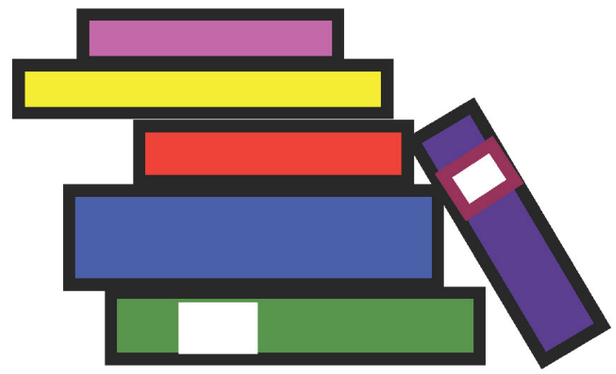
we want students to realize that multiplication describes situations involving equal groups
we want students to notice sizes of groups when deciding whether or not to use multiplication
we want students to see that sometimes rearranging groups can change the way we describe them
eg. 7 + 9 can be rearranged to 8 + 8 which is a double - which might help us calculate a sum





What different addition sentences might tell how many books will be on each shelf after putting away the books on the floor?

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This picture is about addition as combining.
Questions on adding as combining should focus on what the numbers in the addition sentence represent, the fact that the total or sum is greater than either number being added if more than 0 is added.
This picture provides conversations about the variety of number sentences possible. If all 6 books were added to one shelf, sentences such as 3+6 = 9, 5+6=11, or 7+6 = 13. If the additional books were split among shelves sentences such as 3+2 = 5 could be discussed

questions

- Why do you know there will be no totals less than 3?
- What might one greatest total be? How do you know?

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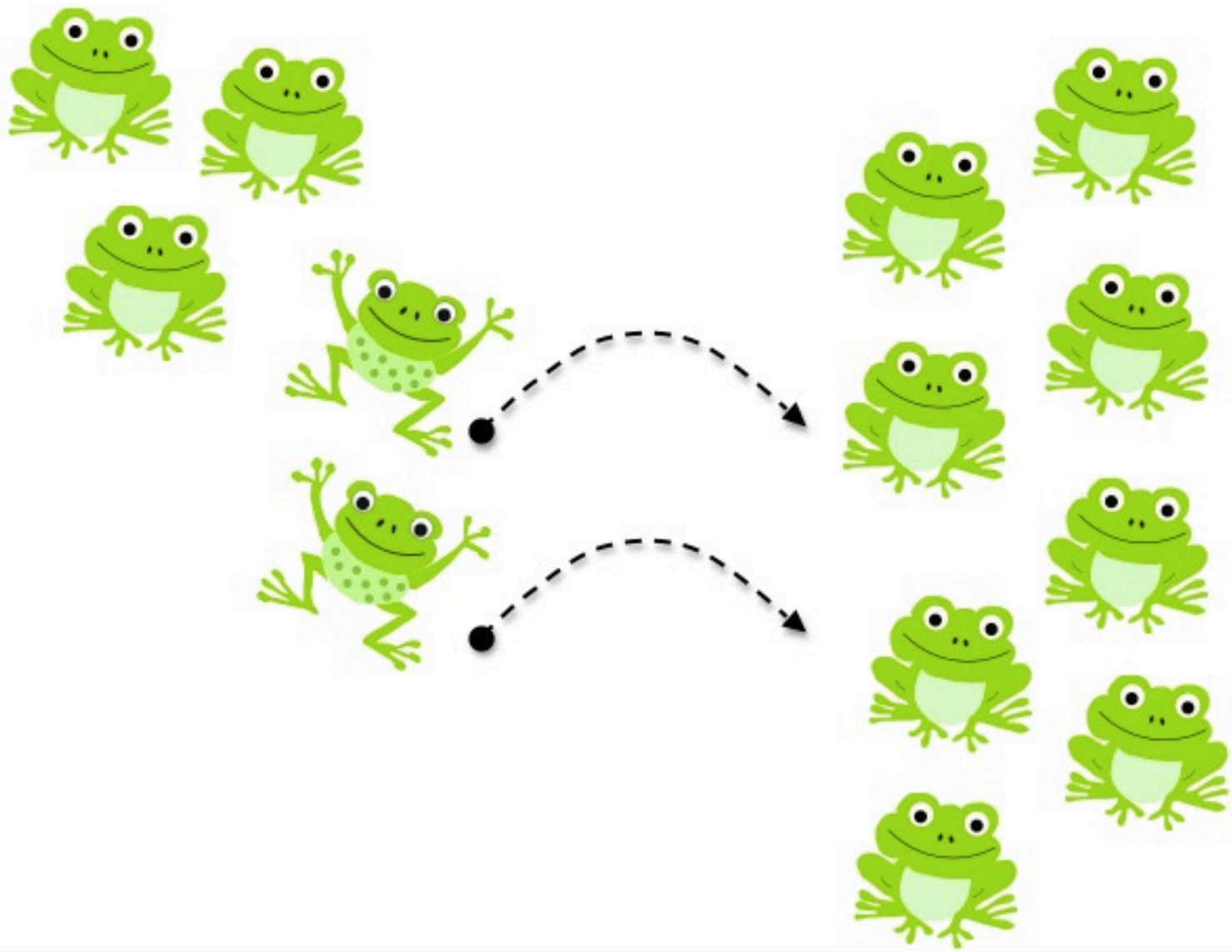
We want students to realize that a sum must be at least as great as the addends when the addends describe objects that can be counted
We want students to practice combining some numbers
We want students to realize that the structure of the problem allows for many possibilities

How are the number sentences you write to tell about all the frogs the same and different after the two frogs move over?



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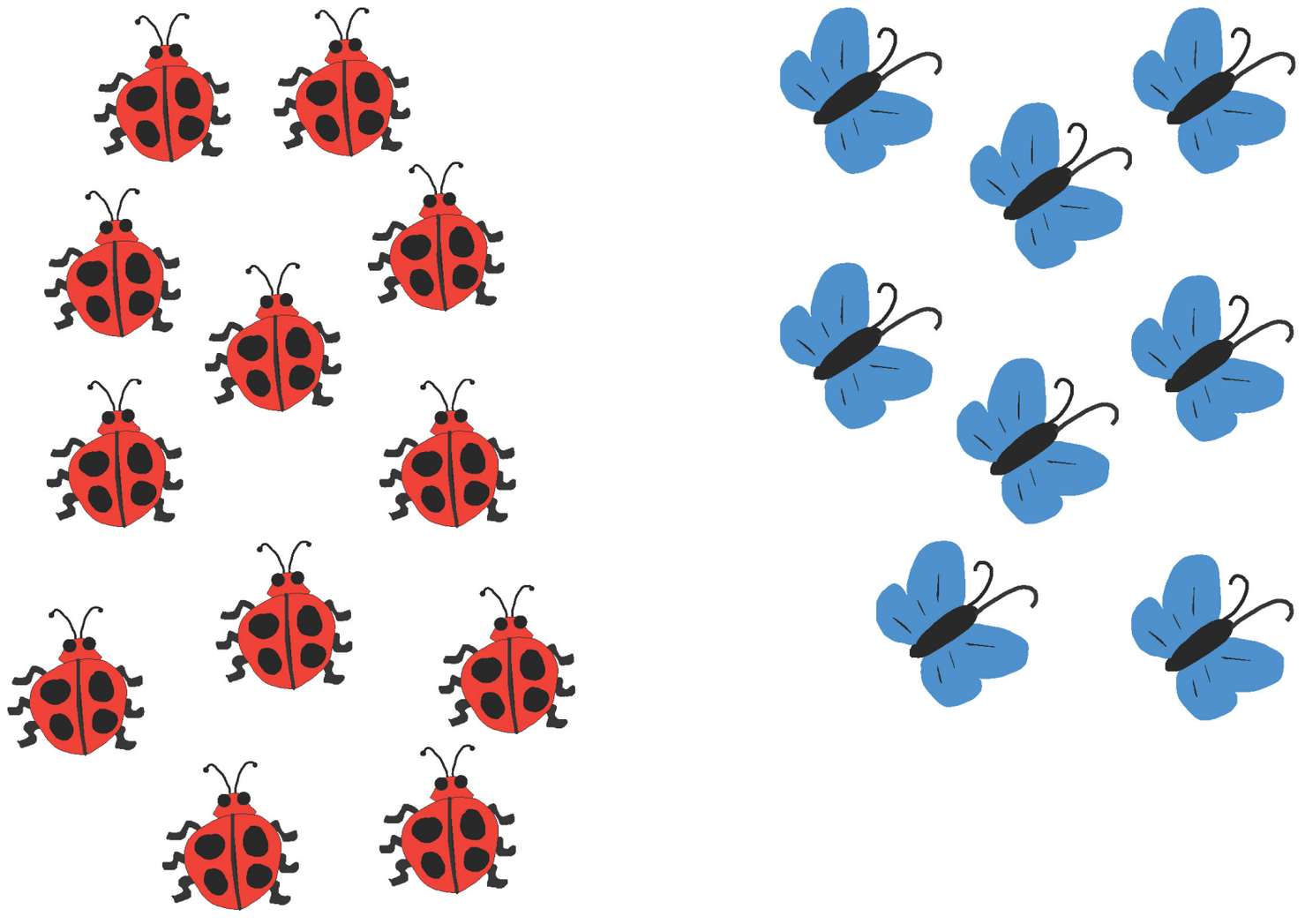
changing addends, but not the sum

picture shows that the reduction of one addend by the same amount by which the other is increased is really a reorganization of an amount, and the total does not change

8+5 = 10 + 3

questions

- What other sentences could you write about the groups of frogs that would be true?
- How would the sentence change if 4 frogs moved over?
- Why did the sum have to be the same for both sentences?
- Why do 5 + 8 and 3 +10 have the same answer?



What does 12 - 8 tell you about the insects?

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Different types of subtraction - where most students in early grades are exposed to subtraction - but knows it as “take away”. It is important for them to know we use subtraction to compare items and to decide how much more one amount is than another.

In this case - we look at how many more ladybugs than butterflies there are. The picture of ladybugs and butterflies is deliberately arranged as it is so that students themselves can recognize that putting the creatures into 1-1 correspondence would help them answer the question

Questions

- Where do you see 12 in the picture?
- Where do you see 8?
- Why do you think a subtraction sentence was used?
- When you take 8 away from 12, you see the 8 items within the 12 items. Why does it makes sense to show all 8 + 12 (or 20) items to compare the ladybugs to the butterflies?

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we want students to associate the pats of the number sentence with the appropriate parts of the real life problem
we want students to recognize that subtraction can tell how much more one amount is than another
we want students to understand why the models for take away subtraction and comparison might be different

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video of Ben - doing subtraction question using visual

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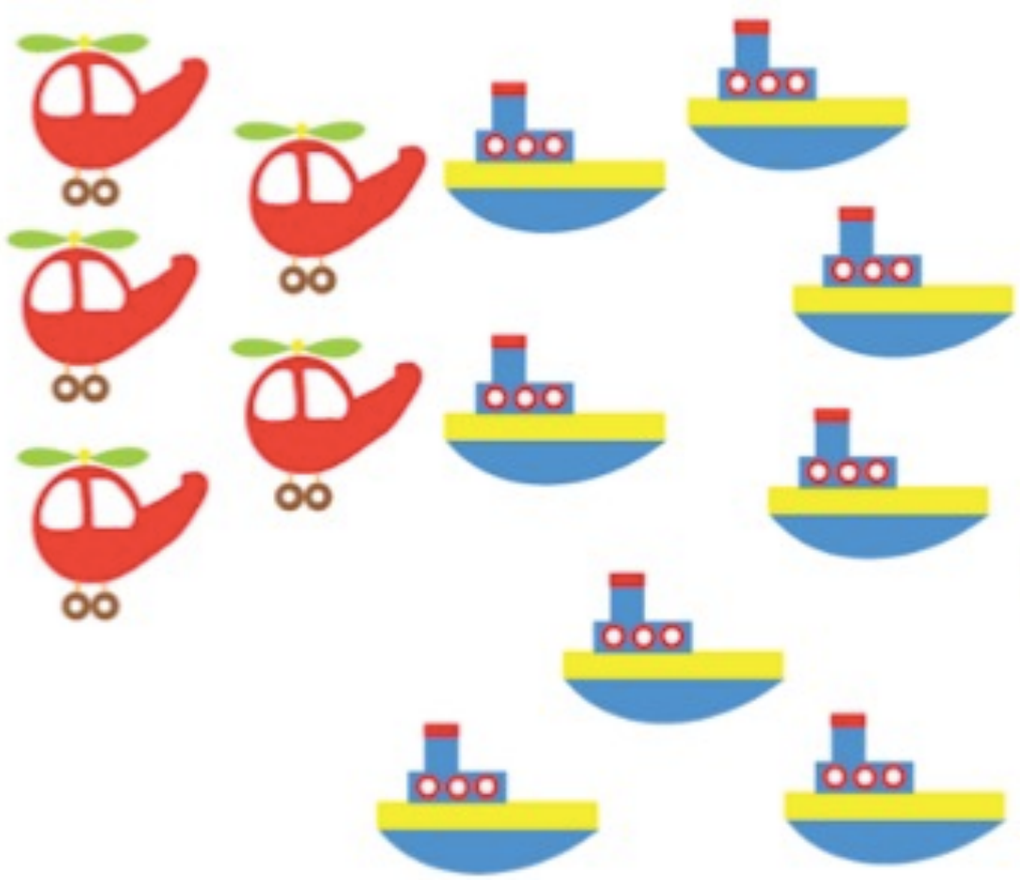
video of Ben - doing subtraction question using visual



Does this picture show addition, subtraction or both?

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relating addition and subtraction

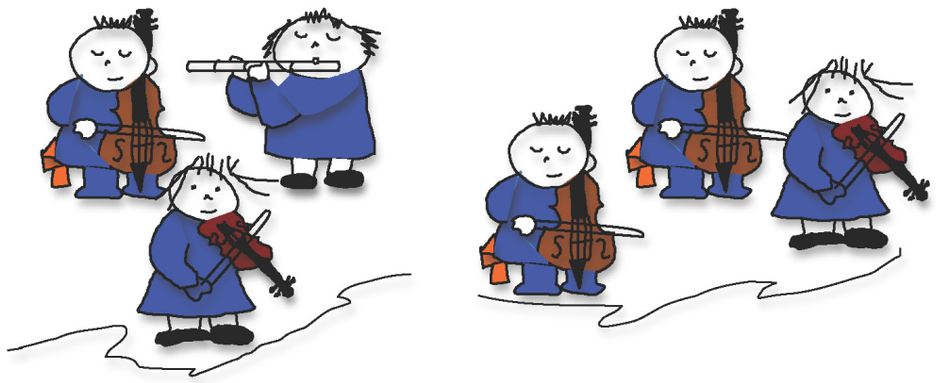
questions

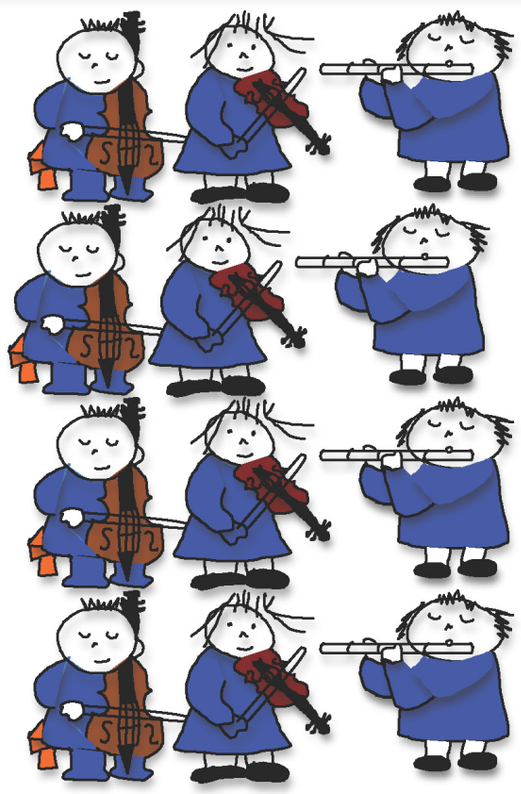
- What addition sentences describe the picture?
- What subtraction sentences describe the picture?
- What do all of the sentences you wrote have in common?

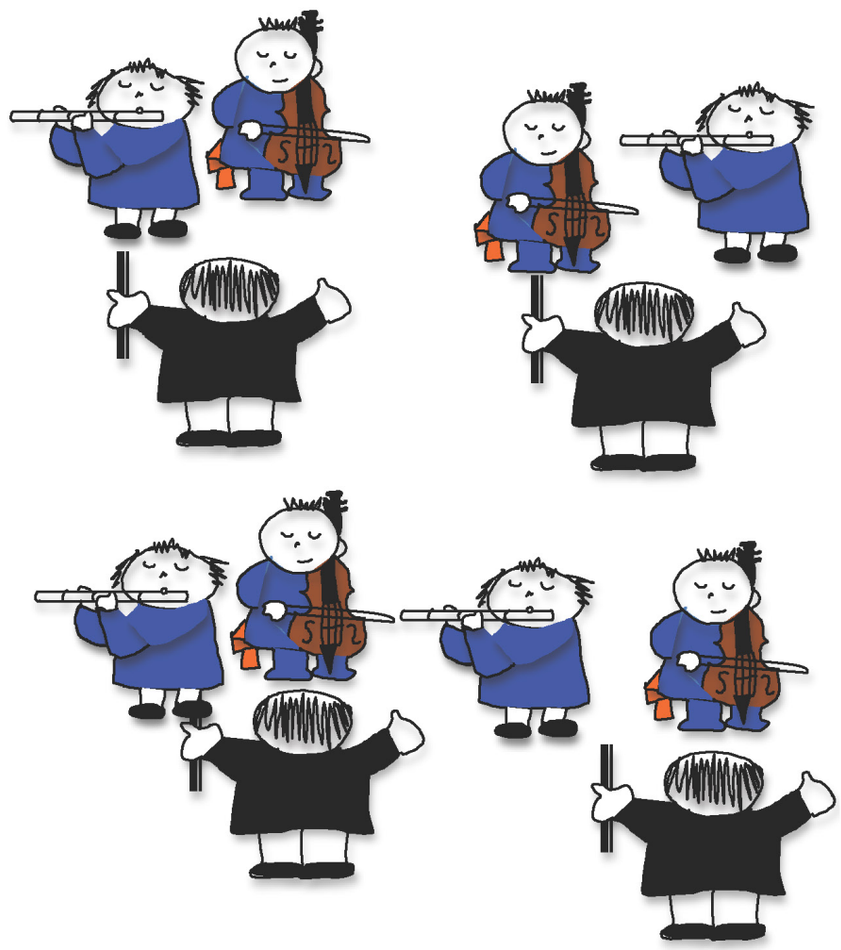
Which pictures make it easy to see that

3 x 4 = 4 x 3?

Which don't?







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This picture is designed to contrast three different situations - one where an array is used so that commutativity of multiplication is very clear (since 4 rows of 3 is clearly 3 columns of 4) and one where it is not too difficult to pull out 3 groups of 4 (the conductors, flue players, and cellists), even thought the visual really only shows 4 groups of 3 and one where it is more challenging to find the 3 groups of 4 among the 4 groups of 3

Questions

- What does the 4 tell you about each picture?
- What does the 3 tell you about each picture?
- How are the pictures alike? Different?

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We want students to realize that a factor could be the number of groups or the size of a group
We want students to see that there are many ways to represent 4 groups of 3
we want students to see that some visual representations of mathematical ideas make it easier to see principles than do other representations

This puddle is 10 steps wide.

This puddle is 16 steps wide.

Who is right?

Questions

- How did you think the girl decided the width of the puddle was 16 steps wide?
- Could both people be right?
- Are steps a good unit to use to measure length?

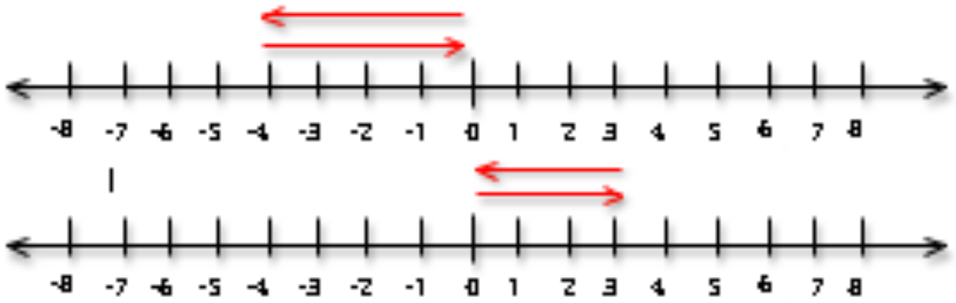
we want students to think about the unit used when measuring and how it would be used
we want students to know that measurements that sound different can both be correct - for example 12 inches and 1 feet could describe the same item.
we want students to know that nonstandard units might not always be as meaningful to a person as standard units

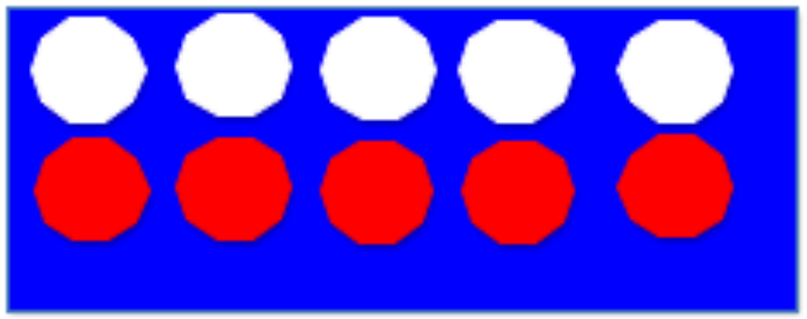
Write the fraction of the red trapezoid that is covered by blue.

Write the fraction of the blue that makes up the red.

What happened and why?

Recipricols





What do all these pictures show?

integers

zero principle

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integer video

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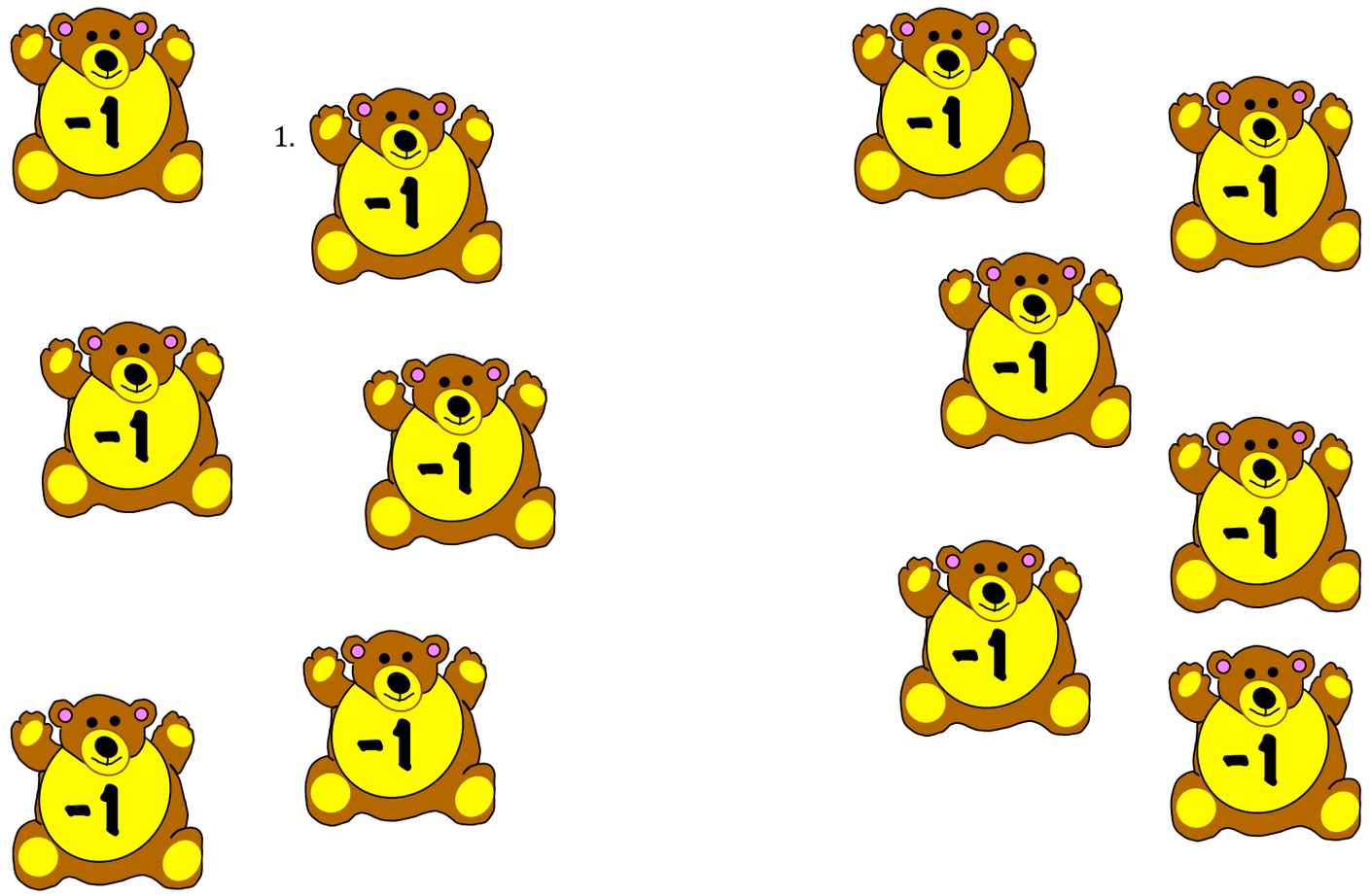
integer video

integers



What multiplication and division sentences does the picture show?

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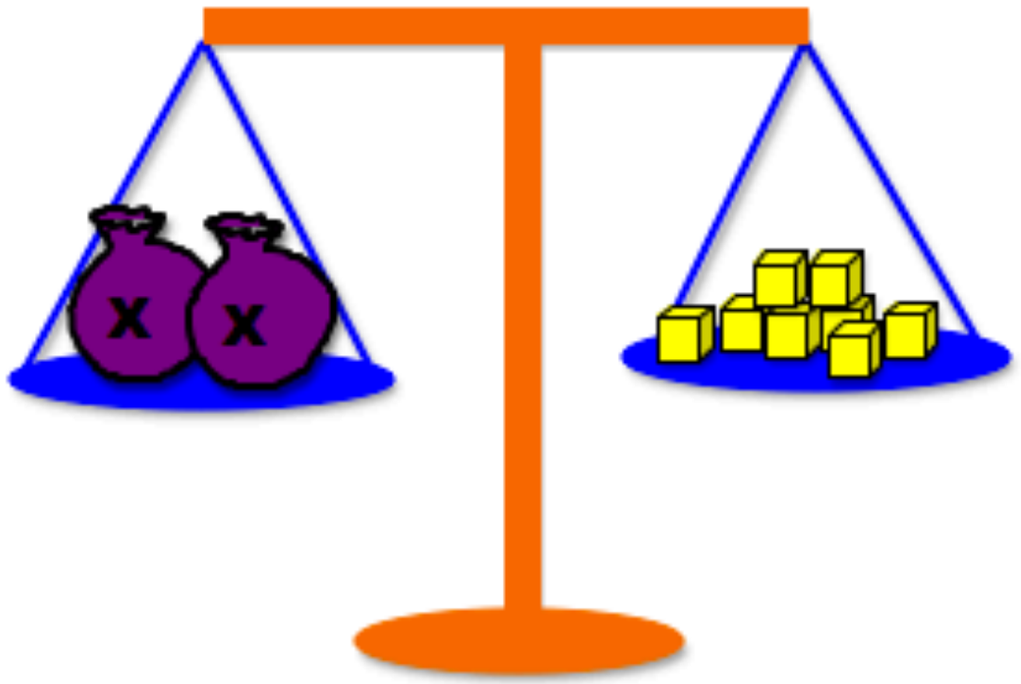
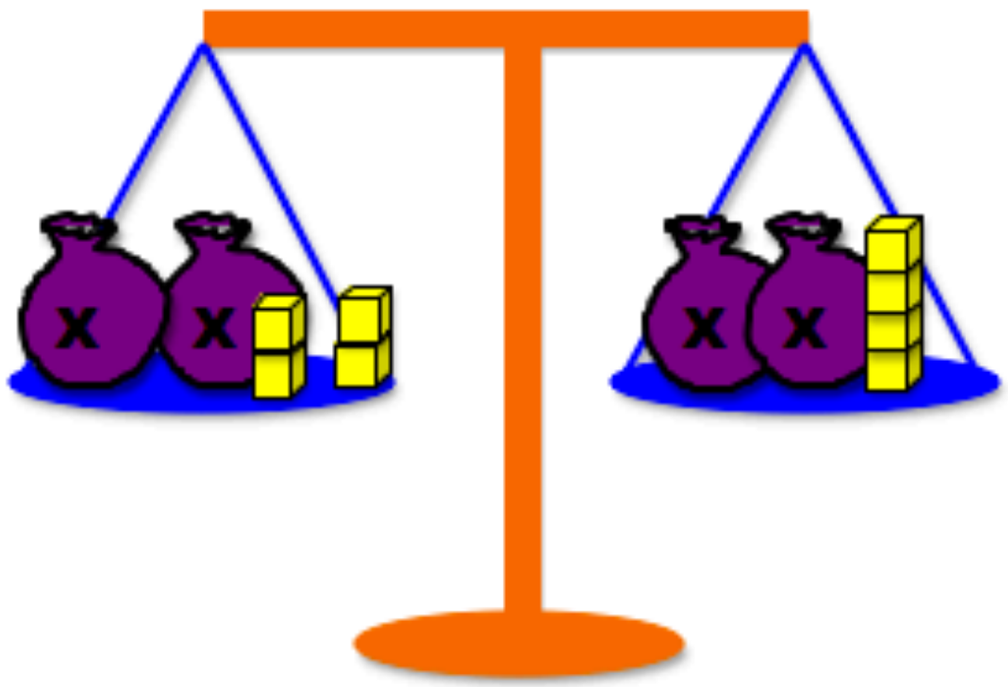
This picture is intended to emphasize the connection between multiplication and division
a students will see 2 x (–6) = –12 as two groups of –6 totalling –12
or –12 divided by 2 = –6 – because –12 can be split into two groups of –6
and –12 divided by –6 = +2 – because there are 2 groups of –6 in –12





How are the equations for the two balances different?

Identity - eg.

$2(x+2) = 2x+4$

Single solution - eg.

$2x = 8$

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equation types

questions

- What equation matches each pan balance?
- What if you put 4 cubes in each bag?
- How do you know that you can't make the first balance work if you put more than 4 cubes in the x bags?
- Which equation tells you that two expressions are equivalent? Which doesn't?

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We want students to relate the physical representations to algebraic equations
we want students to consider the effect of substituting particular values for x in each type of equation
we want students to consider why the first equation has no other solutions
we want students to recognize that if an equation equates two equivalent expressions, it will have an infinite number of solutions but that other equations are valid only for limited values

You know that the yellow arrow is a little longer than the blue one.

Both are whole number amounts.

What could ? be? How do you know?

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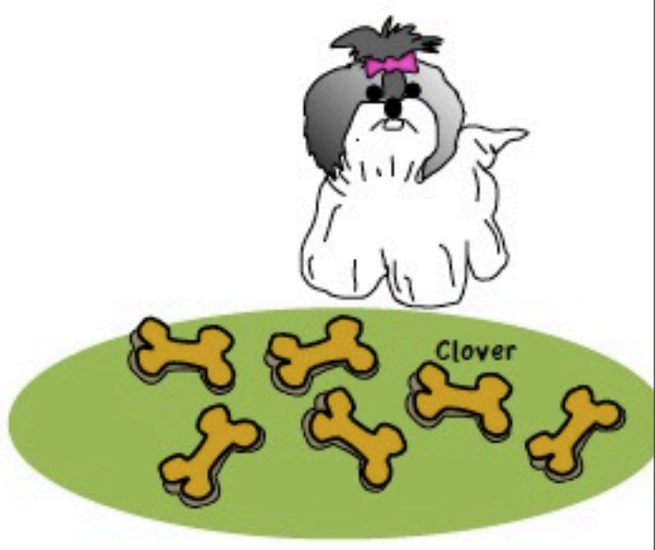
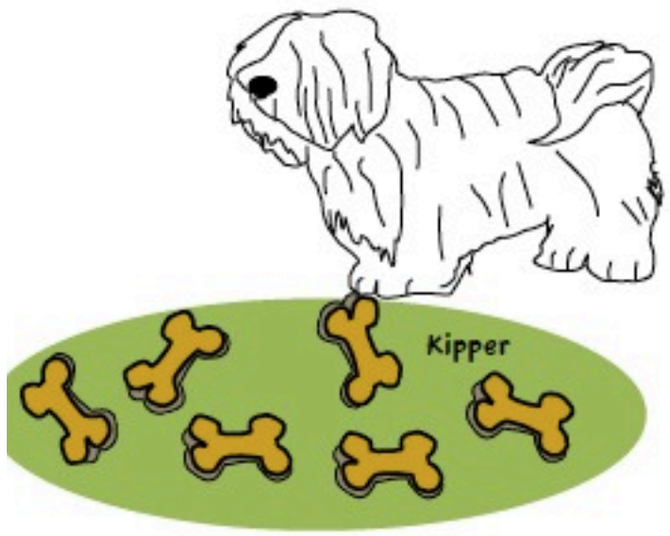
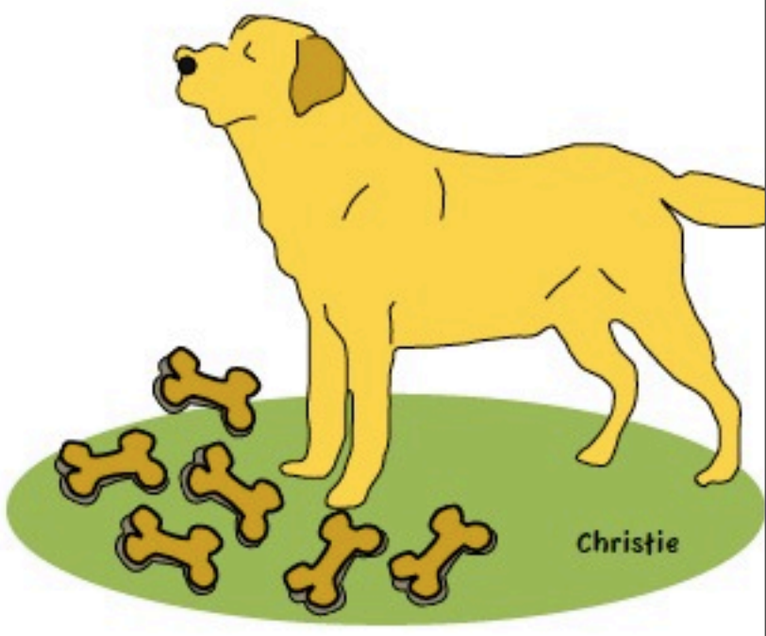
Try this question. (idea from Marian Small)
What did you notice about the amount at “?” ?
The picture is a great way to chose distributive property - clearly - because you can see the 3 blues and 3 yellows.
and the ? is always a multiple of 3.

Factors

How do you know that 6 dogs could also share 24 bones fairly?







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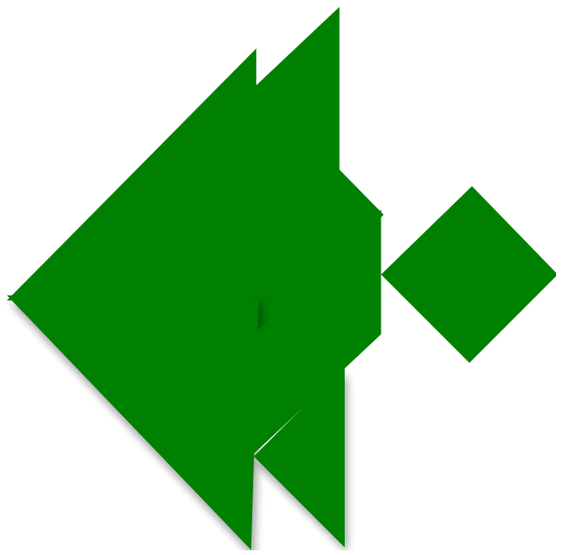
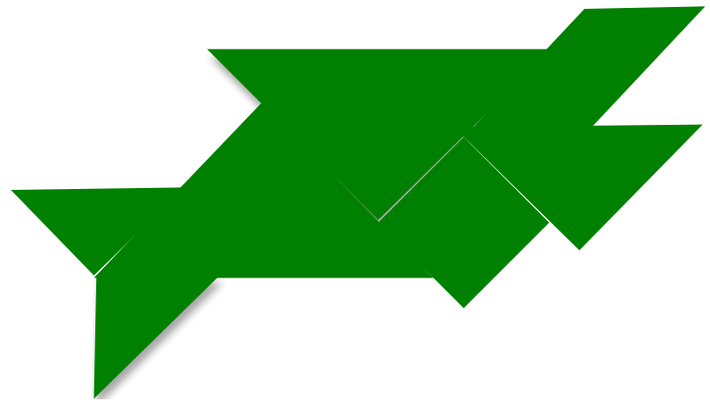
As students try to factor numbers, it helps them to realize that factors come in pairs.
This picture is designed to enable students to see that if 24 is shared among 4, antoehr factor is the share size, which is 6.
This is because division has both a sharing and a how many groups meaning.

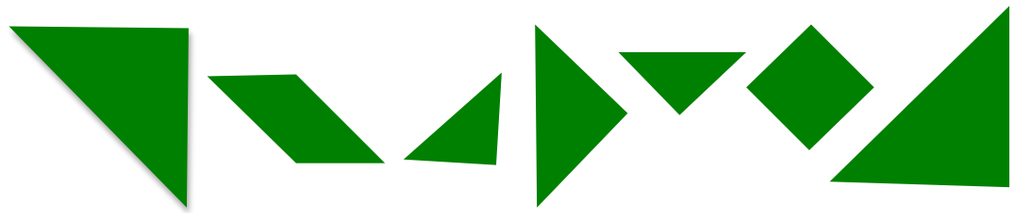
questions

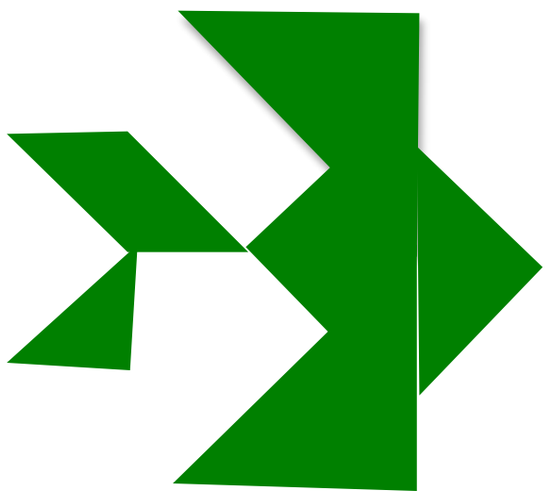
- What other number of dogs could share 24 bones fairly?
- How did the picture help you figure this out?
- Why does getting one factor of a number automatically give you another one?
- Would the factors still come in pairs if there had been 25 bones?

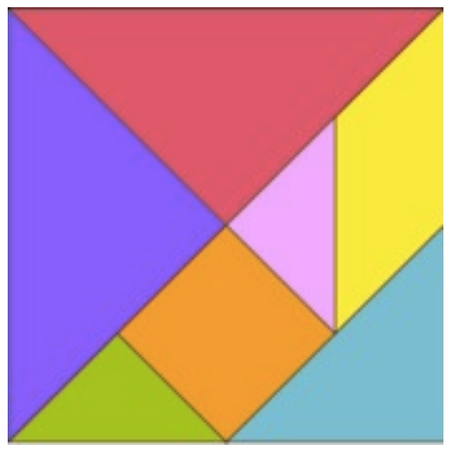
questions geared to help students see that we can go back and forth between these two kinds of meaning











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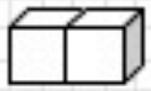
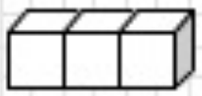
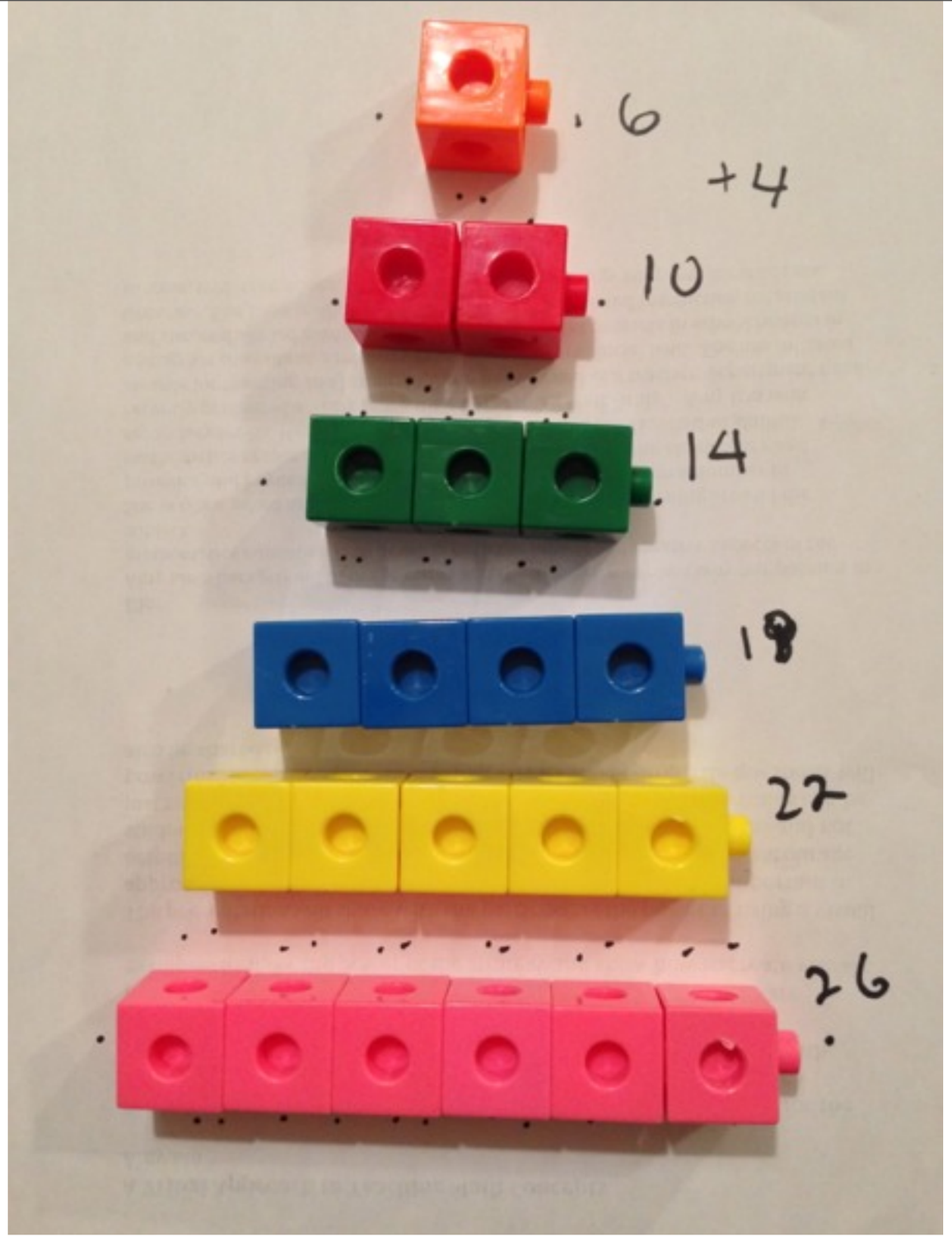
using puzzles help students experience the composition of shapes
however when they use pattern blocks or tangrams, as shown below, to make objects, they are more likely aware that they are composing shapes to make other shapes

questions

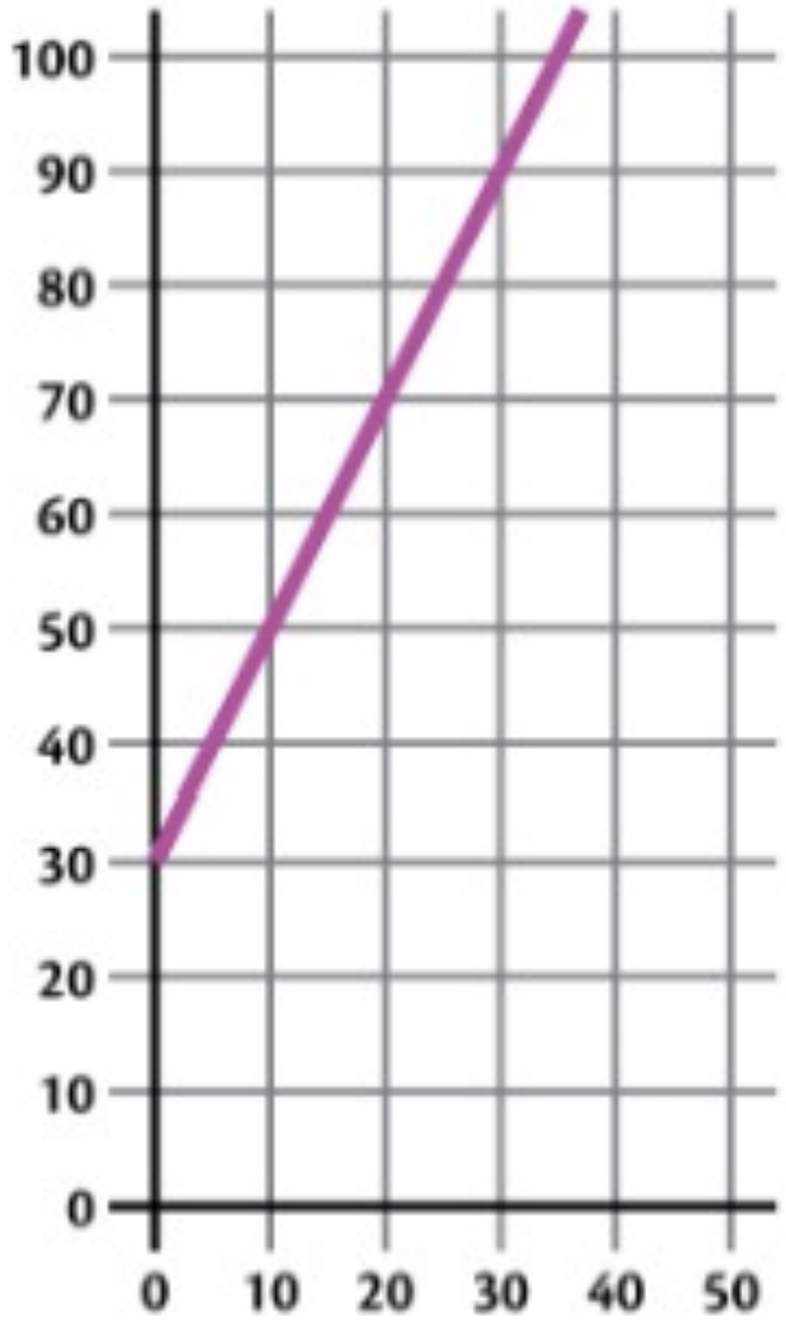
- What shapes do you see making up each picture?
- What does each picture look like?
- Do you think that it would be harder to have different designs if all of the small shapes we used were identical?

Shape	Number of Cubes	Surface Area	First Difference
	1	6	
	2	10	
	3		
	4		
	5		
	6		

Describing Relationships



using linking cubes to visualize 4n+2



The graph tells how much someone might spend if he or she buys a number of $2 plants and a $30 planter. Where on the graph would one find the information from the thee different pictures? Why is the graph a line?

questions

- How could you predict the cost of a purchase of one $30 item and a number of $2 items?
- What table of values would describe the situation in the picture?
- Why might the equation of the line be y = 30 + 2x, if x tells how many $2 plants are purchased?

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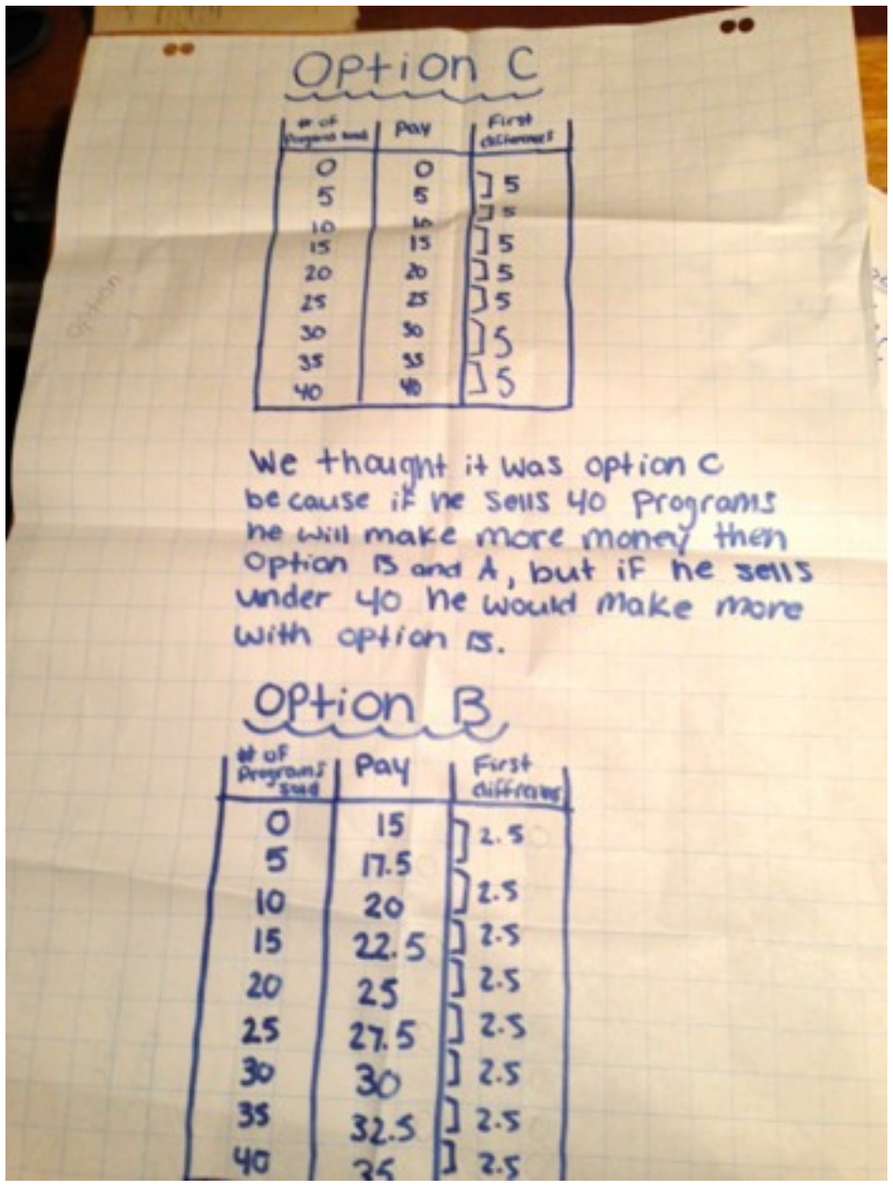
we want students to build the connection between this situation and the ones in the picture by using the number of $2 items purchased as the x value and looking at the line for the corresponding y value



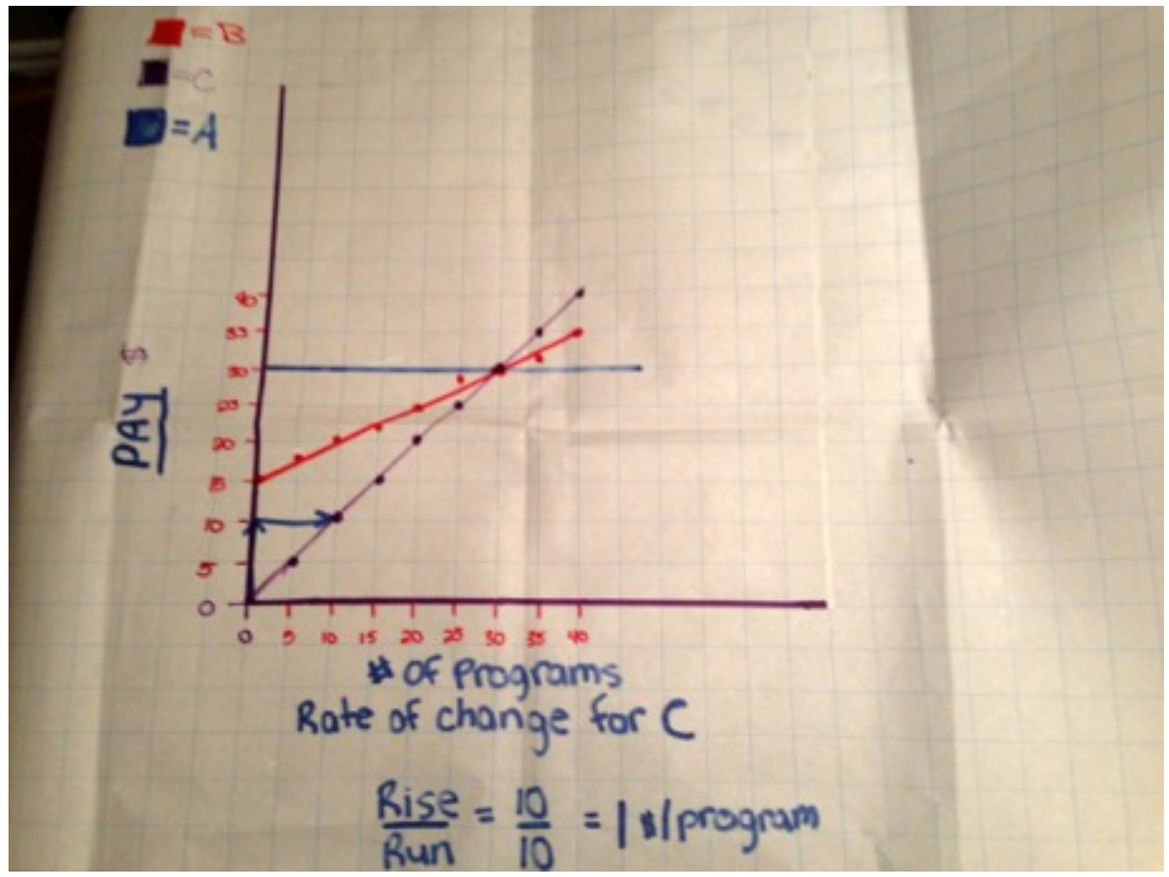
Option C

# of Programs sold	Pay	First differences
0	0	5
5	5	5
10	10	5
15	15	5
20	20	5
25	25	5
30	30	5
35	35	5
40	40	

We thought it was option C because if he sells 40 programs he will make more money then option B and A, but if he sells under 40 he would make more with option B.

Option B

# of Programs Sold	Pay	First differences
0	15	2.5
5	17.5	2.5
10	20	2.5
15	22.5	2.5
20	25	2.5
25	27.5	2.5
30	30	2.5
35	32.5	2.5
40	35	2.5



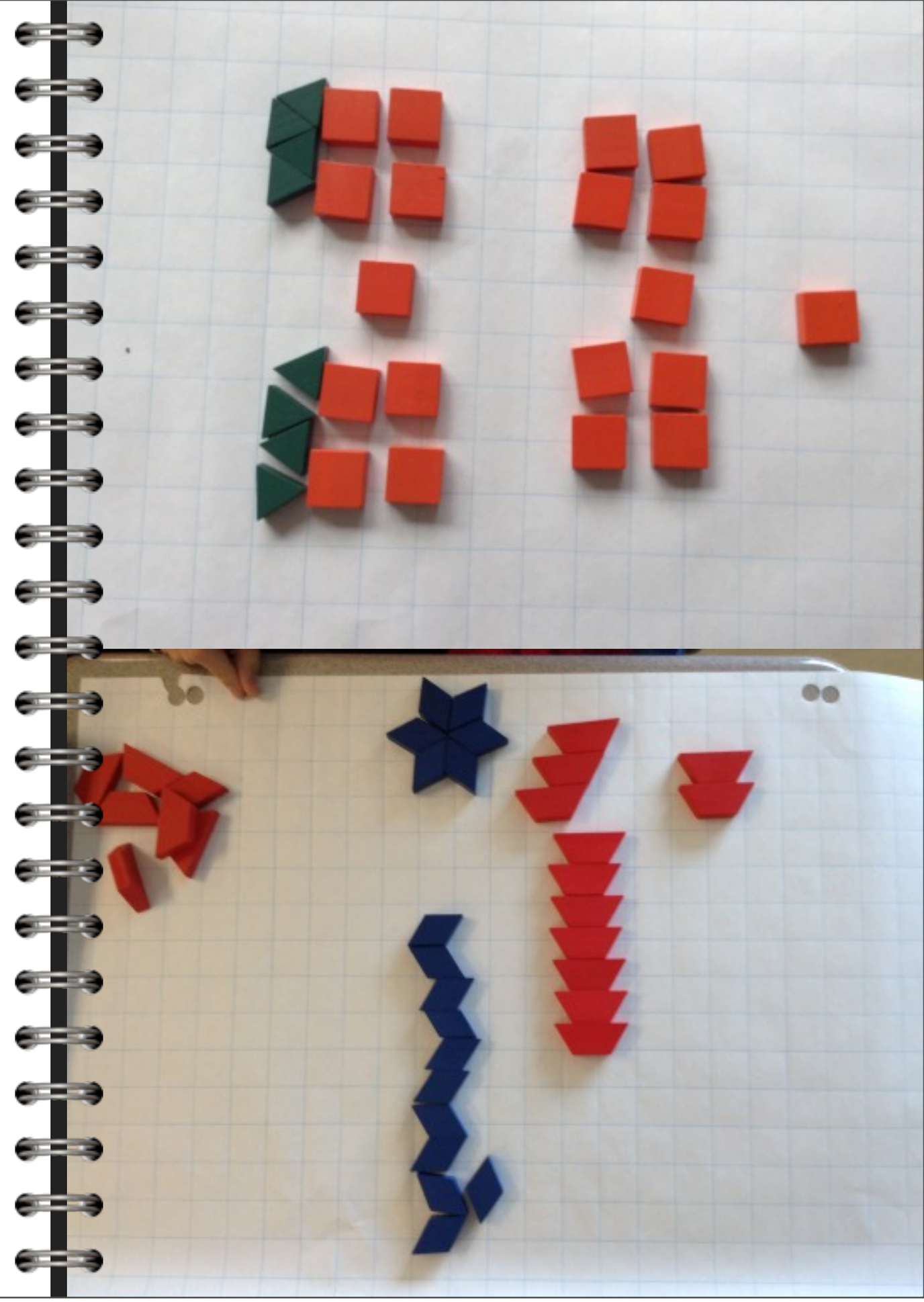
Max just got a job selling programs during Toronto Raptor games. He is offered three options to be paid for his services:

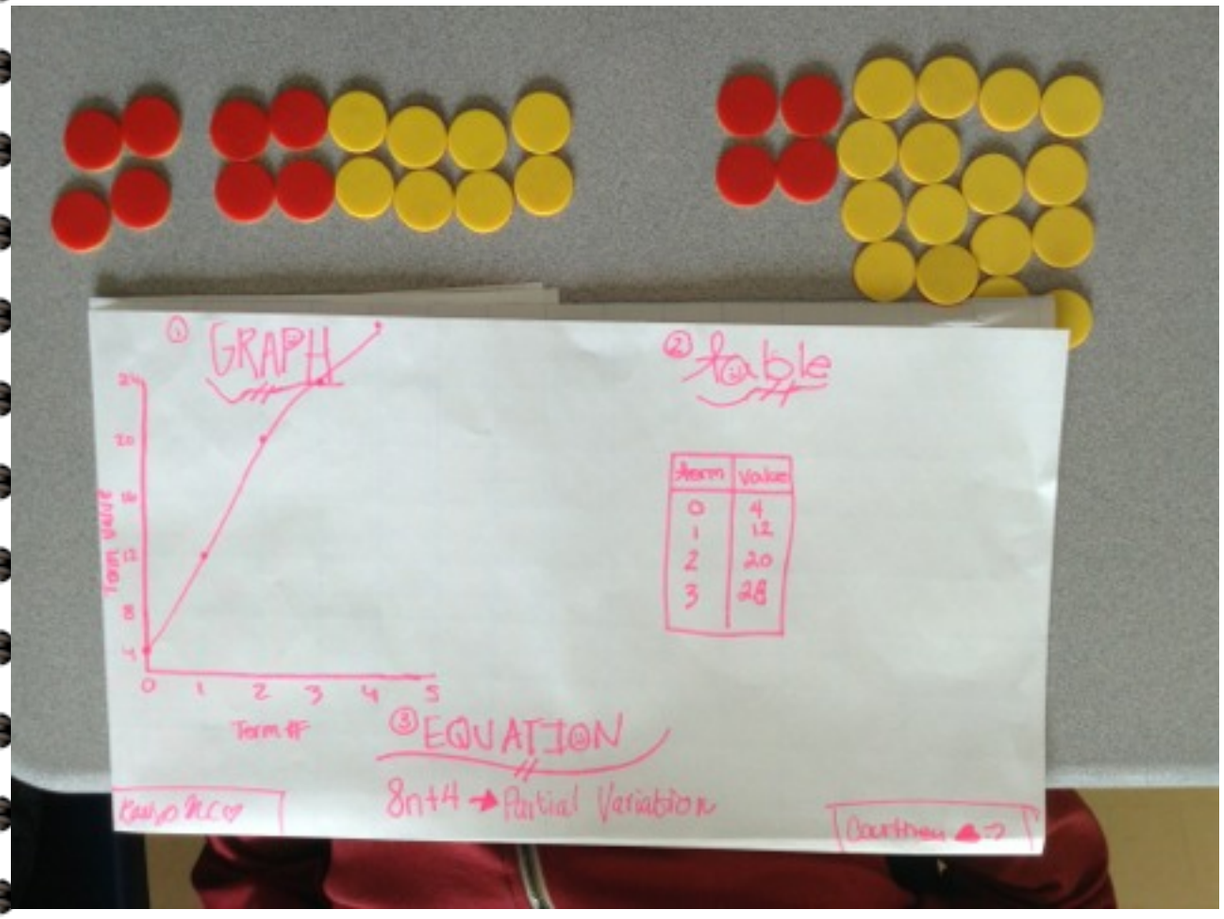
Option A - $30.00 fixed pay

Option B - $15.00 pay plus $0.50 for every program he sells

Option C - $1.00 for every program he sells

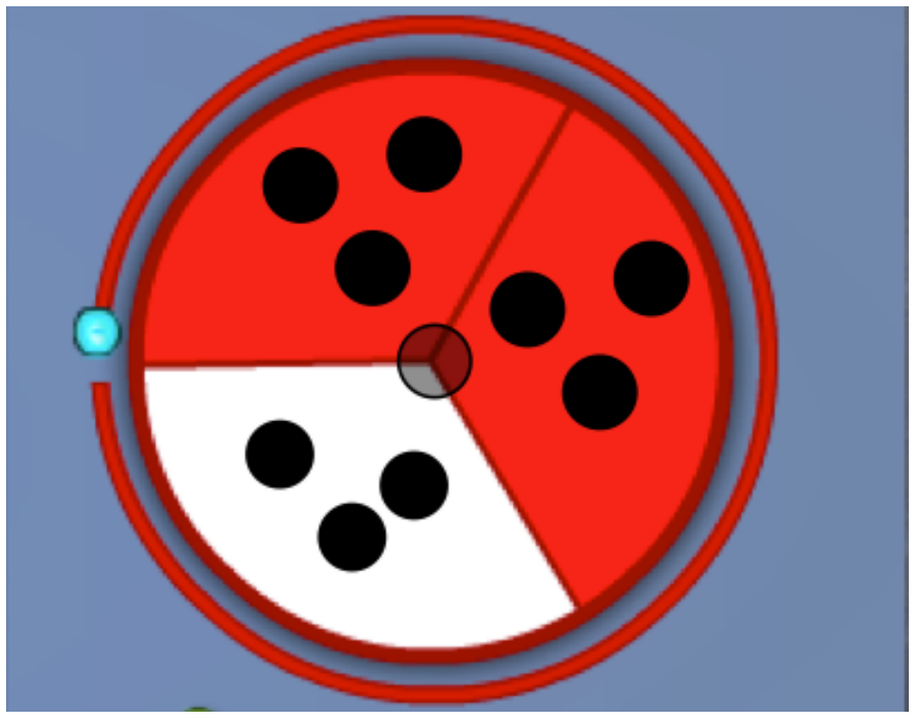
In pairs, use tiles to create a pattern that results in a line graph with a slope of 8 using the term number as the x-value and the term value as the y-value.




① GRAPH
② Table
Term Value
Term #
0 1 2 3 4 5
Term | Value
0 | 4
1 | 12
2 | 20
3 | 28
③ EQUATION
8n+4 → Partial Variation

Create your own

- Choose a topic
- Think of an idea you want to bring out
- Choose a visual to make it happen



What do you know about 2/3 from this picture?

For example:

Idea - part of a set and part of a whole meanings are related

Thank you!

amylin62@me.com

Tedx Sixteen Mile Creek

Teaching Math-ish-ly